

Chairs Days : Insurance, Actuarial Science, Data and Models

Valuing Life as an Asset, as a Statistic and at Gunpoint

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1. Introduction

Motivation and outline

TABLE : Comparison HK value and VSL (in \$)

	Average HK life value HK	Average VSL VSL
Poor	249 532	2 719 261
Fair	318 865	5 126 530
Good	388 198	7 239 006
Very Good	457 531	9 518 831
Excellent	526 864	11 864 750
Mean	420 729	3 351 519
Median	457 731	8 803 507
<i>Empirical literature</i>		
	€ [300, 900]K\$ [Huggett and Kaplan, 2016]	€ [4.2, 13.7]M\$ [Robinson and Hammitt, 2016]

VSL is **10-20** times larger than the HK value of life!...How can we explain and assess this large discrepancy of valuation methods? ▶

Main research questions and contributions

- # 1 Can we provide a reasonable metric for the value of life against which the two alternatives can be gauged?

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- ✓ Provide **common** theoretical framework for HK, WTP, GPV and VSL.
- ✓ **Closed-form solutions** for HK, WTP, VSL and GPV values of life to evaluate :
 - ▶ Role of preferences, technological, distributional parameters.
 - ▶ Role of wealth, human capital
 - ▶ Shape of WTP.
 - ▶ Aggregation issues.

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 - ▶ Aggregation issues.
- ✓ **Structurally** estimate WTP, three values with **common** data set.

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- ✓ VSL is appropriate when computing a collective value on small *indiscriminate* reductions on mortality for which society will ultimately end up paying the costs (e.g., public's safety);

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4 Other implications : modeling of stochastic life cycle models, mortality/morbidity/financial risks, longevity risks.

Road map

1. Introduction
2. A common framework for life valuation
3. Values of life
4. Structural estimation
5. Discussion
6. Conclusion

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such that the probability of death by age t (death risk exposure) is monotone increasing in λ_m :

$$\begin{aligned} \mathcal{P}(t) &= \Pr (T_m \leq t) \\ &= 1 - \exp(-\lambda_m t) \end{aligned}$$

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- ▶ Changes in death risk exposure $\mathcal{P} \Leftrightarrow$ changes in the instantaneous death intensity λ_m

► Law of motion H_t

$$dH_t = [I_t^\alpha H_t^{1-\alpha} - \delta H_t] dt - \phi H_t dQ_{st}$$

where dQ_{st} is a Poisson depreciation (morbidity) shock with constant intensity λ_{s0} that further depreciates the health stock by $\phi \in (0, 1)$.

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- **Budget constraint and income** : Individuals can trade in two risky assets to smooth out shocks to consumption—stock and insurance *against* health depreciation

$$\begin{aligned} dW_t &= [rW_t + Y_t - c_t - I_t] dt + \pi_t \sigma_S [dZ_t + \theta dt] \\ &\quad + x_t [dQ_{st} - \lambda_{s0} dt], \\ Y_t &= y + \beta H_t, \end{aligned}$$

where π_t denotes the risky portfolio and x_t the units of an actuarially-fair insurance.

Preferences

Stochastic Differential Utility (Duffie and Epstein, 1992) :

- ▶ Disentangle risk aversion γ from intertemporal elasticity of substitution ε ;
- ▶ Minimum subsistence consumption a ;
- ▶ Preference for life over death ;
- ▶ $V^m \equiv 0$;

$$U_t = E_t \int_t^{T_m} \left(f(c_\tau, U_\tau) - \frac{\gamma |\sigma_\tau(U)|^2}{2U_\tau} \right) d\tau,$$

where the age of death T_m is the first occurrence of a Poisson process with constant intensity λ_m and the Kreps-Porteus aggregator is :

$$f(c_t, U_t) = \frac{\rho U_t}{1 - 1/\varepsilon} \left(\left(\frac{c_t - a}{U_t} \right)^{1 - \frac{1}{\varepsilon}} - 1 \right).$$

Optimal allocation V, c, I, π, x

Theorem

Optima closed-form allocations are given by:

$$c_t = a + A(\lambda_m)N(W_t, H_t)$$

$$\pi_t = \frac{\theta}{\gamma\sigma_S}N(W_t, H_t)$$

$$x_t = \phi P(H_t)$$

$$I_t = \left(\alpha^{\frac{1}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}}\right) P(H_t)$$

$$V_t(W_t, H_t, \lambda_m) = \Theta(\lambda_m)N(W_t, H_t)$$

$$\checkmark \underbrace{N(W, H)}_{\text{Net worth}} = \underbrace{W}_{\text{Fin. Wealth}} + \underbrace{(y - a)/r}_{\text{NPV of fixed inc. stream}} + \underbrace{P(H)}_{\text{Shadow value = BH}}$$

$$\checkmark \text{ Marginal value of } N : \Theta(\lambda_m) = \tilde{\rho}A(\lambda_{m0})^{\frac{1}{1-\varepsilon}} \geq 0$$

$$\checkmark \text{ MPC : } A(\lambda_m) = \varepsilon\rho + (1 - \varepsilon)(r - \lambda_m + 0.5\theta^2/\gamma) \geq 0$$

3. Values of life

Human capital value of life

Proposition

The HK value of life $v_{h,t} = v_h(W_t, H_t, \mathcal{P}_0)$ is the expected discounted present value over stochastic horizon T_m of labor revenue flows, net of investment costs,

$$\begin{aligned}v_{h,t} &= E_t \int_0^{T^m} m_{t,\tau} [Y(H_\tau^*) - I_\tau^*] d\tau \\ &= E_t \int_0^{T^m} m_{t,\tau} [y + (\beta H_\tau^* - I_\tau^*)] d\tau\end{aligned}$$

where $m_{t,\tau} = m_\tau / m_t$ with $m_t = \exp(-rt - \theta Z_t - 0.5\theta^2 t)$, and writes

$$v_h(H, \lambda_m) = C_0 \frac{y}{r} + C_1 P(H)$$

$$\text{with } C_0 = \frac{r}{r + \lambda_m} \quad \text{and} \quad C_1 = \frac{r - (\alpha B)^{\frac{\alpha}{1-\alpha}}}{r + \lambda_m - (\alpha B)^{\frac{\alpha}{1-\alpha}}}.$$

Comparison

- ✓ Reduced-form model : Assume a constant interest rate r and net dividends are $Y(H) - I = y + Y_n(H)$, then

$$v_h = \frac{r}{r + \lambda_m} \times \frac{y}{r} + \frac{r - g_n}{r + \lambda_m - g_n} \times \frac{Y_n(H)}{r - g_n}$$

where g_n is the growth rate of Y_n .

- ✓ Structural model :

$$v_h(H, \lambda_m) = \frac{r}{r + \lambda_m} \times \frac{y}{r} + \frac{r - (\alpha B)^{\frac{\alpha}{1-\alpha}}}{r + \lambda_m - (\alpha B)^{\frac{\alpha}{1-\alpha}}} \times P(H).$$

Willingness to pay

Definition

The willingness to pay $v = v(W, H, \mathcal{P}_0, \Delta)$ to avoid a permanent change $\Delta \in [\mathcal{P}_0, 1 - \mathcal{P}_0]$ in death risk exposure \mathcal{P} solves

$$V(W - v, H, \mathcal{P}_0) = V(W, H, \mathcal{P}_0 + \Delta).$$

- ✓ $\Delta > 0$: Indifference between paying the equivalent variation $v > 0$ at base risk and not paying but facing higher death risk
- ✓ $\Delta < 0$: Indifference between receiving compensation $-v > 0$ and foregoing lower death risk exposure.

Proposition

The willingness to pay to avoid an admissible change $\Delta \in \mathcal{A}_m$ is :

$$v(W, H, \lambda_m, \Delta) = \left[1 - \frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)} \right] N(W, H)$$

an increasing and concave function of Δ that is bounded by :

$$\inf_{\Delta \in \mathcal{A}_m} v(W, H, \lambda_m, \Delta) = \left[1 - \frac{\Theta(0)}{\Theta(\lambda_m)} \right] N(W, H)$$
$$\sup_{\Delta \in \mathcal{A}_m} v(W, H, \lambda_m, \Delta) = N(W, H)$$

with $\lambda_m^* = \lambda_m + \delta$.

Value of a statistical life

Proposition

The value of a statistical life $v_s = v_s(W, H, \mathcal{P}_0)$ is the negative of the MRS between the probability of death and wealth computed from the indirect utility evaluated at base risk \mathcal{P}_0 (or λ_m) :

$$v_s = - \left. \frac{V_{\mathcal{P}}(W, H, \mathcal{P})}{V_W(W, H, \mathcal{P})} \right|_{\mathcal{P}=\mathcal{P}_0}.$$

and is given by

$$v_s(W, H, \lambda_m) = \frac{1}{A(\lambda_m)} N(W, H)$$

where $A(\lambda_m)$ is the MPC and N the net total wealth.

Equivalently, the VSL is also the marginal willingness to pay :

$$v_s(W, H, \mathcal{P}_0) = \left. \frac{\partial v(W, H, \mathcal{P}_0, \Delta)}{\partial \Delta} \right|_{\Delta=0} = \lim_{\Delta \rightarrow 0} \frac{v(W, H, \mathcal{P}_0, \Delta) - v(W, H, \mathcal{P}_0, 0)}{\Delta}.$$

Theoretical VSL vs Empirical VSL

Definition

The empirical value of a statistical life, $v_s^e = v_s^e(W, H, \mathcal{P}_0, \Delta)$ is given by :

$$v_s^e(W, H, \mathcal{P}_0, \Delta) = \frac{v(W, H, \mathcal{P}_0, \Delta)}{\Delta}$$

for small increment $\Delta = 1/n$ where n is the size of the population considered.

- ▶ As $\Delta \rightarrow 0$, $v_s^e(W, H, \mathcal{P}_0, \Delta) \simeq v_s(W, H, \mathcal{P}_0)$;
- ▶ The bias $v_s^e - v_s$ depends on the curvature of the WTP and Δ .

Gunpoint value of life

Proposition

The gunpoint value $v_g = v_g(W, H, \mathcal{P}_0)$ is the WTP to avoid certain, instantaneous death and it solves :

$$V(W - v_g, H, \mathcal{P}_0) = V^m$$

where V^m is the utility at death, and is given by

$$v_g(W, H) = N(W, H) \equiv W + \frac{y - a}{r} + BH.$$

Implications

- ✓ Using the homogeneity properties of the SDU (foregone utility = foregone excess consumption), the NTW (GPV) is the expected discounted present value of excess consumption ($c_t - a$) along the optimal path :

$$v_g(W, H) = E_t \int_t^{\infty} m_{t,\tau} \bar{c}_\tau d\tau = N(W, H)$$

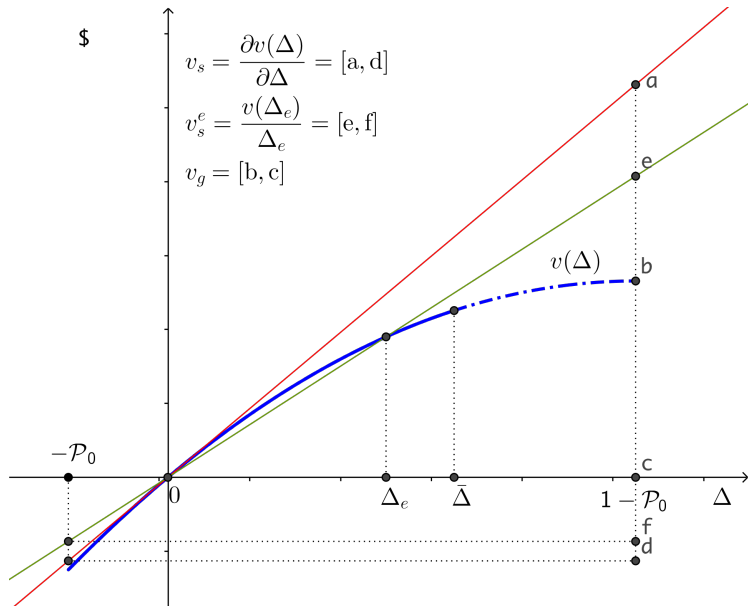
- ✓ Unless y/r is large, $v_g(W, H) - v_h(W, H, \lambda_m) \geq 0$;
- ✓ GPV is the admissible upper bound of the WTP to avoid a change in death risk exposure :

$$v_g(W, H) = \sup_{\Delta \in \mathcal{A}_m} v(W, H, \lambda_m, \Delta)$$

- ✓ $v_g(W, H) = A(\lambda_m) v_s(W, H, \lambda_m) < v_s(W, H, \lambda_m)$;
- ✓ $g(c_t - a) = g(v_{s,t}) = g(v_{g,t})$ where $g(z)$ denotes the growth rate of z .

To summarize

return



4. Structural estimation

▶ Econometric model

$$\mathbf{Y}_j = \mathbf{B}(\theta)\mathbf{X}_j + \mathbf{u}_j$$

where

$$\mathbf{Y}_j = [c_j, \pi_j, x_j, l_j, Y_j]'$$

$$\mathbf{X}_j = [1, W_j, H_j]$$

▶ Data : PSID 2013

- ✓ Health : "Poor" to "Excellent" using self-reported status (household head).
- ✓ Financial wealth = risky (stocks in publicly held corporations, mutual funds, investment trusts, private annuities, IRA's or pension plans) plus riskless assets (checking accounts plus bonds plus remaining IRA's and pension).

Estimation of structural parameters

Parameter	Value	Parameter	Value
a. Law of motion health			
α	0.6843 (0.3720)	δ	0.0125 (0.0060)
ϕ	0.0136 ^c		
b. Sickness and death intensities			
λ_s	0.0347 (0.0108)	λ_m	0.0283 (0.0089)
d. Preferences			
γ	2.8953 (1.4497)	ε	1.2416 (0.3724)
a	0.0140 ^c	ρ	0.0500 ^c

Value of Statistical Life vs HK Value

Wealth quintile level	Health level				
	Poor	Fair	Good	Very Good	Excellent
a. Value of Statistical Life v_s					
1	2 167 573	4 379 551	6 591 529	8 803 507	11 015 485
2	2 168 877	4 380 874	6 593 136	8 805 188	11 017 133
3	2 188 829	4 400 253	6 614 190	8 827 429	11 040 023
4	2 360 907	4 582 287	6 800 733	9 021 052	11 238 999
5	4 710 118	7 889 684	9 595 444	12 136 981	15 012 108
All					
- mean			8 351 519		
- median			8 803 507		
b. Human Capital Value of Life v_h					
	251 968	323 127	394 287	465 446	536 606
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- mean			437 756		
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Gunpoint Value of Life vs HK Value

Wealth quintile level	Health level				
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	a. Gunpoint Value of Life v_g				
1	116 121	234 620	353 120	471 619	590 119
2	116 191	234 691	353 206	471 709	590 207
3	117 259	235 729	354 334	472 901	591 433
4	126 478	245 481	364 327	483 274	602 093
5	252 329	422 664	514 045	650 199	804 225
All					
- mean			447 405		
- median			471 619		
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Extensions

- ✓ Endogenous mortality and morbidity [Overview](#)

$$\lambda_m(H_{t-}) = \lim_{\tau \rightarrow 0} \frac{1}{\tau} P_t [t < T_m \leq t + \tau] = \lambda_{m0} + \lambda_{m1} H_{t-}^{-\xi_m}$$
$$\lambda_s(H_{t-}) = \eta + \frac{\lambda_{s0} - \eta}{1 + \lambda_{s1} H_{t-}^{-\xi_s}}$$

- ✓ Ageing : Time-varying parameters $\lambda_{m,t}$, $\lambda_{s,t}$, ϕ_t , δ_t or β_t .
- ✓ SHARE data
- ✓ Immortal Life Value : WTA a compensation to renounce to perpetual life [Overview](#)

Results remain applicable and are robust.

Different valuation methods to evaluate the price of human life

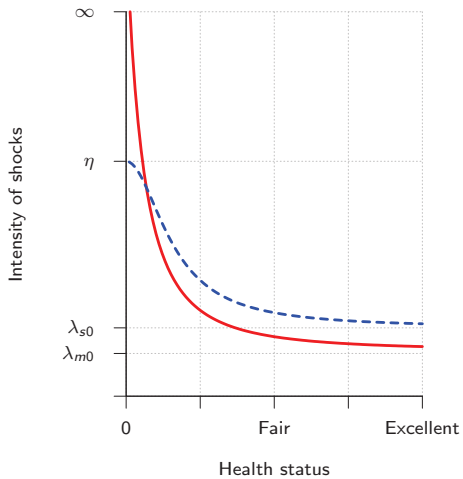
- ▶ **Human capital life value** : Prejudice caused to society by the death/injury of an individual (occupational, end-users' wrongful death litigation)—Present value of the net cash flow associated with human capital (asset pricing view)

$$v_{h,t} = E_t \sum_{s=0}^{T_m} \left(\frac{1}{1+r} \right)^s D_{t+s}$$

where D_{t+s} denotes the net dividend at time $t + s$ —marketed labor income *minus* all expenses to maintain human capital.

return

FIGURE : Intensities of mortality and morbidity shocks



Consequences

- ▶ First-order approximation around the exogenous mortality and morbidity-based model ;
- ▶ First-order correction for the endogenous mortality/morbidity :

$$\begin{aligned} N_1(W, H) &= N_0(W, H) - \lambda_{s1} H^{-\xi_s} I_s P_0(H) \\ &= W + \frac{y - a}{r} + \underbrace{HB [1 - \lambda_{s1} H^{-\xi_s} I_s]}_{P_1(H)} \end{aligned}$$

Return

Immortal Life Value, ILV

