An Introduction to Moral Hazard in Continuous Time

Dylan Possamaï

Columbia University, NY

Chairs Days: Insurance, Actuarial Science, Data and Models, June 12th, 2018

Outline

Motivation

B. Salanié, The economics of contracts

Customers know more about their tastes than firms, firms know more about their costs than the government and all agents take actions that are at least partly unobservable.

- Vast economic literature revisiting general equilibrium theory by incorporating incitations and asymmetry of information (Holmström, Hart, Milgrom, Mirrlees, Picard, Tirole, Rogerson, Salanié, Laffont, Martimort, Bolton, Dewatripoint, Ekeland, Rochet, Choné, Carlier, Williams, Sung, Cvitanić, Zhang, Sannikov,...).

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- Moral hazard: situation where an Agent can benefit from an action (unobservable), whose cost is incurred by others $\longrightarrow$ a type of externality
- How can one design "optimal" contracts?

Relevance to insurance

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- Better identification of clients' actuarial risk is paving the way for innovative contracts, with targeted prevention and virtuous incentive mechanisms.
- Crux of the matter: information sharing $\longrightarrow$ auto insurance (pay how you drive), health insurance (connected objects)...
- How to quantify and exploit optimally the information shared by the clients?

Modelisation

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- Principal has the initiative of the contract.
- Agent can accept or refuse it, but is then fully committed.
- When he accepts, Agent has to perform a costly task (examples to come).

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- Choice of Agent impacts the **distribution** of an output process X

$$X_t = X_0 + \int_0^t \lambda_s(X_{\cdot\wedge s}, \nu_s)ds + \int_0^t \sigma_s(X_{\cdot\wedge s}, \nu_s)dW_s^\nu, \ t \in [0, T],$$

where W^ν is a $\mathbb{P}^\nu-$Brownian motion.

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where W^ν is a $\mathbb{P}^\nu-$Brownian motion.

- Profit of Principal depends on X, which he observes. But ν is inaccessible! $\Longrightarrow$ hidden action $\Longrightarrow$ asymmetry of information.

Examples (from finished or on-going works)

• Optimal remuneration of managers (Élie, Mastrolia, Réveillac and Villeneuve)

$$\begin{cases} X \longrightarrow \text{value of the firm.} \\ \nu \longrightarrow \text{work of the manager.} \end{cases} \qquad X_t = X_0 + \int_0^t \nu_s ds + \sigma W_s^\nu.$$

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- Contract $\longrightarrow$ salary of the manager.

Examples (from finished or on-going works)

- Delegated portfolio management (Cvitanić and Touzi).

$$\begin{cases} X \longrightarrow \text{value of the portfolio.} \\ \nu \longrightarrow \text{investment choices.} \end{cases} \qquad X_t = X_0 + \int_0^t \nu_s \cdot (b_s ds + \sigma_s dW_s^{\nu}).$$

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- Contract $\longrightarrow$ remuneration of the fund manager.

Examples (from finished or on-going works)

• Electricity demand response management (Aïd, Élie, Hubert, Mastrolia and Touzi).

$$\begin{cases} X \longrightarrow \text{deviation from baseline consumption.} \\ \nu = (\alpha, \beta) \longrightarrow \text{effort on the mean and the volatility of consumption.} \end{cases}$$

$$X_t = X_0 - \int_0^t \alpha_s \cdot \mathbf{1} ds + \int_0^t \mathrm{diag}(\sigma) \sqrt{\beta_s} \cdot dW_s^\nu.$$

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- Contract $\longrightarrow$ compensation of the consumer.

Examples (from finished or on-going works)

• Optimal securitization of mortgage loans (Pagès, Hernández Santibáñez and Zhou).

$$\begin{cases} X \longrightarrow \text{number of observed defaults of the } I \text{ loans.} \\ \nu \longrightarrow \text{bank monitoring actions (number of loans non-monitored).} \end{cases}$$

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• Contract $\longrightarrow$ remuneration of the bank, and liquidation procedure.

Examples (from finished or on-going works)

- Insurance contracts (Kazi-Tani, Hernández Santibáñez and Zhou).

$$\begin{cases} X \longrightarrow \text{number of accidents.} \\ \nu = (\lambda, \alpha, \beta) \longrightarrow \text{effort to reduce frequency and severity of accidents.} \end{cases}$$

X process with jump compensator $\rho_s^\nu(dx, ds) = \lambda_s \delta_{Y_s}(dx)ds$, with

$$dY_s = \alpha_s ds + \sigma\sqrt{\beta_s} dW_s^\nu.$$

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- Contract $\longrightarrow$ premium charged to the insureds, and deductible.

Agent's problem

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- Agent solves then

$$V_0^A(\xi) := \sup_{\nu} \mathbb{E}^{\mathbb{P}^\nu}\Bigg[\underbrace{\xi(X_{\cdot \wedge T})}_{\text{Salary}} - \int_0^T \underbrace{c_t(X_{\cdot \wedge t}, \nu_t)}_{\text{Cost}}\, dt\Bigg].$$

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- Dependence of ξ on the whole trajectory of X is **crucial**.
- Agent faces a non–Markovian stochastic control problem.

Principal's problem

Principal looks for a Stackelberg equilibrium and proceeds in 2 steps.

(*i*) Compute best reaction function of Agent to $\xi \longrightarrow \nu^\star(\xi) \longrightarrow \mathbb{P}^\star(\xi)$.

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(i) Compute best reaction function of Agent to $\xi \longrightarrow \nu^\star(\xi) \longrightarrow \mathbb{P}^\star(\xi)$.

(ii) Feedback optimisation on the contracts

$$V_0^P := \sup_{\xi \in \Xi_R} \mathbb{E}^{\mathbb{P}^\star(\xi)}\Big[U(X_{\cdot\wedge T}, \xi(X_{\cdot\wedge T}))\Big],$$

- U: utility function of Principal.
- Ξ_R: contracts such that $V^A(\xi) \geq R$ (participation constraint).

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- Agent's problem: non–Markovian control $\longrightarrow$ (2)BSDEs.
- Principal's problem: non–standard optimisation over random variables $\longrightarrow$ necessary reinterpretation to pave the way to explicit solutions and, at least, to **efficient numerical methods!**

Outline

Intuition and verification

- Dynamic Agent's problem writes

$$V_t^A(\xi) := \underset{\nu}{\operatorname{essup}}\ \mathbb{E}^{\mathbb{P}^\nu}\left[\xi(X_{\cdot\wedge T}) - \int_t^T c_s(X_{\cdot\wedge s}, \nu_s)ds \middle| \mathcal{F}_t\right],\ t \in [0,T].$$

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- Hamiltonian naturally associated to this control problem is

$$H(t, x, z, \gamma) := \sup_v h(t, x, z, \gamma, v),$$

$$h(t, x, z, \gamma, v) := \lambda_t(x, v) \cdot z + \frac{1}{2}\mathrm{Tr}\left[\gamma(\sigma\sigma^\top)_t(x, v)\right] - c_t(x, v).$$

Intuition and verification

- If $V_t^A(\xi) =: v^A(t, X_{\cdot\wedge t})$ is a "smooth" function of $(t, X_{\cdot\wedge t})$, where smooth means that an Itō formula is valid

$$dv^A(t,X) = \partial_t v^A(t, X_{\cdot\wedge t})dt + Z_t \cdot dX_t + \frac{1}{2}\mathrm{Tr}\left[\Gamma_t d\langle X\rangle_t\right],$$

with the correspondence

$$Z_t \longrightarrow "\partial_x v^A(t, X_{\cdot\wedge t})", \quad \Gamma_t \longrightarrow "\partial_{xx} v^A(t, X_{\cdot\wedge t})".$$

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- Z and Γ only supposed integrable $\longrightarrow$ Sobolev type regularity.
- Dynamic programming and martingale optimality principle $\Longrightarrow$

$$\partial_t v^A(t, X_{\cdot\wedge t}) + H\big(t, X_{\cdot\wedge t}, Z_t, \Gamma_t\big) = 0.$$

Intuition and verification

- To sum up

$$V_0^A(\xi) = \xi + \int_0^T H\big(t, X_{\cdot\wedge t}, Z_t, \Gamma_t\big)\,dt - \frac{1}{2}\int_0^T \mathrm{Tr}\,[\Gamma_t d\langle X\rangle_t] - \int_0^T Z_t\cdot dX_t.$$

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- It is a non–linear representation theorem for ξ! In addition, optimal effort of Agent is given by

$$\nu_t^\star \in \underset{v}{\mathrm{argmax}}\left\{h(t, X, Z_t, \Gamma_t, v)\right\}.$$

Intuition and verification

The converse statement is true! It is a simple verification theorem

Theorem

If, for an "admissible" pair (Z, Γ) and $V_0 \in \mathbb{R}$, the contract $\xi_T^{V_0,Z,\Gamma}$, with

$$\xi_t^{V_0,Z,\Gamma} := V_0 - \int_0^t H(s, X, Z_s, \Gamma_s)ds + \frac{1}{2}\int_0^t \mathrm{Tr}\left[\Gamma_s d\langle X\rangle_s\right] - \int_0^t Z_s \cdot dX_s,$$

is offered to Agent, then his utility is V_0 and his optimal effort

$$\nu_t^\star \in \underset{v}{\mathrm{argmax}}\left\{h(t, X, Z_t, \Gamma_t, v)\right\}.$$

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However, *quid* of the existence of Z and Γ for general contracts?

2BSDEs

- Existence of Z is classical (standard representation theorem), but less clear for Γ. Introduce

$$F(t,x,z,v) := \sup_{\gamma} \left\{ \frac{1}{2}\mathrm{Tr}[\gamma(\sigma\sigma^\top)_t(x,v)] - H(t,x,z,\gamma) \right\}.$$

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- We have for any control ν

$$V_0^A(\xi) = \xi - \int_0^T F(t,X,Z_t,\nu_t)dt - \int_0^T Z_t \cdot dX_t + K_T^\nu,$$

where K^ν is non–decreasing with

$$K_T^\nu := \int_0^T F(t,X,Z_t,\nu_t)dt - \frac{1}{2}\mathrm{Tr}[\Gamma_t d\langle X\rangle_t] + H(t,X,Z_t,\Gamma_t)dt.$$

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2BSDEs

- Relaxation of existence of $\Gamma \iff K^{\nu}$ not necessarily absolutely continuous.
- The triplet $(V^A(\xi), Z, (K^{\nu})_{\nu})$ solves then exactly a 2BSDE with terminal condition ξ and generator F!
- Study of wellposedness of these equations and their generalisations with Élie, Kazi-Tani, Piozin, Mastrolia, Matoussi, Sabbagh, Soner, Tan, Touzi, Zhang and Zhou.

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2BSDEs

- 2BSDEs allow for a general probabilistic representation of the value function of Agent, but do not give in general access to optimal effort!
- Nonetheless, one can prove that the contracts ξ of the form $\xi^{V_0,Z,\Gamma}$ are "dense" in an appropriate class of admissible contracts.
- No loss of generality in restricting to them!

Outline

Principal's problem

- Why is the class $\xi_T^{v,Z,\Gamma}$ useful? $\longrightarrow$ Principal's problem becomes

$$V_0^P = \sup_{v \geq R} V_0^P(v),$$

$$V_0^P(v) := \sup_{Z,\Gamma} \mathbb{E}^{\mathbb{P}^\star(Z,\Gamma)}\Big[U\big(X_{\cdot\wedge T}, \xi_T^{v,Z,\Gamma}\big)\Big],$$

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- $V_0^P(v)$ is value function of a standard stochastic control problem, with state variables X and $\xi^{v,Z,\Gamma}$, and with controls Z and Γ.
- Continuation utility of Agent is a performance index for Principal.
- HJB equation (possibly path–dependent) associated $\longrightarrow$ **explicit solutions or efficient numerical schemes!**

Extensions

- Extension to one Principal and several Agents. Nash equilibrium between Agents characterized by a system of (quadratic) (2)BSDEs $\longrightarrow$ Principal' problem is standard control again with 2$\times$number of Agents state variables (with Élie)

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- Can consider the mean–field game limit where the number of Agents goes to infinity $\longrightarrow$ Principal solves a McKean–Vlasov control problem $\longrightarrow$ HJB equation in infinite dimension (with Mastrolia and Élie)
- Modelization of free rider or ripple effects.

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- More contractual possibilities: several competing Principals, firing or promoting, limited liability, inter–temporal payments, learning . . .

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- More contractual possibilities: several competing Principals, firing or promoting, limited liability, inter–temporal payments, learning ...
- To each extension is associated a control problem (and (2)BSDEs) with different specificities $\longrightarrow$ reflections, constraints, weak terminal conditions, stochastic targets,...

Thank you for your attention!