Tables de mortalité *best estimate* : une approche de crédibilité pour les portefeuilles de petite taille

Yahia Salhi $^{\alpha}$ Pierre Thérond $^{\alpha,\beta}$

$^{\alpha}$ISFA - Université Lyon 1

$^{\beta}$Galea & Associés

21/03/2017

CHAIR OF EXCELLENCE
Data Analytics & Models for Insurance
BNP PARIBAS CARDIF
LABORATOIRE SAF SCIENCES ACTUARIELLE & FINANCIERE

1. Introduction I

- Recently, interest from life insurers to assess their own mortality risk has considerably increased.
- Princing and accounting regulations require insurers to use conservative mortality tables (cf. art. A132-18 C. Ass.).
- New prudential (Solvency II) and Financial Reporting regulations require to estimate an explicit risk margin / risk adjustment and to make an assessment of the mortality without bias.
- This change has a significant impact on the manner to build the life tables for the computation of provisions : An insurance company must
  - directly measure the biometric risk (mortality / longevity) of her own policyholders,
  - reassess these risks on a regular basis.

1. Introduction II

- Following the work of Bühlmann and Gisler (2005) and Hardy and Panjer (1998), we propose a credibility approach which consists on reviewing, as new observations arrive, the mortality rates based on
    1. Parametric curve
    2. Semi-parametric approach

- Such an adjustment makes possible to add a **structure** in the mortality pattern which is useful when portfolios are of limited size so as to ensure a good representation over the **age-band**.

- It is also beneficial given the heterogeneity in the cost of the guarantees according to age.

1.1. Data Structure and Notation I

- Suppose that we have at our disposal age-specific mortality statistics originating from n portfolios.
- For each portfolio $i \in \{1, \cdots, n\}$, we observe the deaths of exposures over a period T_i and an age band $[\underline{x}, \overline{x}]$
- Denote the number of individuals at attained age x during calendar year $t = 1, \cdots, T_i$ by $L^i_{x,t}$ and $D^i_{x,t}$ represents the number of deaths recorded. We also introduce the following notation,

$$D^i_{x,\bullet} = \sum_{t=1}^{T_i} D^i_{x,t}, \quad E^i_{x,\bullet} = \sum_{t=1}^{T_i} E^i_{x,t}, \quad D^i_{\bullet,t} = \sum_{x=\underline{x}}^{\overline{x}} D^i_{x,t}, \quad E^i_{\bullet,t} = \sum_{x=\underline{x}}^{\overline{x}} E^i_{x,t},$$

- The "•" indexation refers to the summation over the index of interest.

2.1. Mortality Law for Single Portfolios I

When it comes to the study of the mortality at a single portfolio level, some specific issues arise :

Size of populations Insured population are generally of small size, which may bias the estimation of the mortality.

Length of historical data Available age-specific mortality statistics lacks of deepness.

Scale of available data : Insured portfolios show a typical behavior compared to a national mortality. The mortality of insured population is significantly lower than the national population from which it is drawn. National demographic statistics as substitutes are useless as they may not have the same characteristics than the initial population.

2.2. Mortality Law for Single Portfolios I

- We are interested in the behavior, over time, of the mortality (or a proxy) at the portfolio level
- One may think of the n portfolios as a subset of the reference population and thus each population is characterized by a risk profile Θ_i
- The relative trend level of portfolio i with respect to the baseline mortality (reference) is characterized by the risk profile θ_I which is a realization of Θ_i
- Θ_i representing the unobserved characteristics of the portfolio i mortality (with respect to the baseline). We implicitly take into account the heterogeneity of the i's portfolio mortality profile.

Parametric Approach : Makeham Law

3.1. Notation and Assumptions I

Makeham (1867) assumes that the force of mortality ϕ_x at attained age x has the following form :

$$\phi_x = A + BC^x, \quad \text{with } A, B \text{ and } C \text{ are some constants}$$

.

- For the baseline ϕ_x^b the parameters A^b, C^b and B_t^b are estimated using the aggregated data, i.e. $E_{x,\bullet}^{\bullet}$ and $D_{x,\bullet}^{\bullet}$.
- For each portfolio i, the "trend" is incorporated in the parameter B^i denoted B_t^i

3.1. Notation and Assumptions II

- In view of the specific issues discussed earlier, for each portfolio $i \in \{1, \cdots, n\}$ we will make the following assumptions :
  - The baseline mortality μ_x^b is described by the Makeham model.
  - The age effect is similar on the n portfolios. Companies specific model is assumed to share the same parameters $A^i = A^b$ and $C^i = C^b$ for any $i \in \{1, \cdots, n\}$
  - The time-dependent parameter B_t^i is fitted at each period.
  - The unique parameter that captures the specific mortality at the portfolio level is B_t^i, which would a priori not be the same over companies due to the heterogeneity of the underlying populations
- We are interested in the behavior, over time, of the random variable

$$X_t^i = \frac{B_t^i}{B_t^b}.$$

3.1. Notation and Assumptions III

Goal

- In view of the available data up to time T_i, i.e. X_t^i for $t = 1, \cdots, T_i$, we aim to find the *best estimate* of $\mathbb{E}[X_t^i | \Theta_i] = \mu(\Theta_i)$, which is unknown such that $\mathbb{E}[\mu(\Theta_i)]$.
- For this purpose and using the usual credibility setting.

3.1. Notation and Assumptions IV

H0 $\mathbb{V}\text{ar}\big[\mu(\Theta_i)\big] = \mathbb{V}\text{ar}[\Theta_i] := \tau^2$, and $\mathbb{E}[\sigma^2(\Theta_i)] = \mathbb{E}[\Theta_i] := \sigma^2$

H1 Conditionally on Θ_i, X_t^i are independent with moments

$$\mathbb{E}[X_t^i|\Theta_i] = \mu(\Theta_i) \text{ and } \mathbb{V}\text{ar}(X_t^i|\Theta_i) = \frac{\sigma^2(\Theta)}{\omega_t^i}$$

for some functions $\mu(\Theta_i)$ and $\sigma^2(\Theta)$, where

$$\omega_t^i = \frac{\sum_x (C^b)^x L_{x,t}^i}{\sum_{i,x} (C^b)^x L_{x,t}^i},$$

measures the weight given to the period t from the portfolio i.

H2 The pairs (Θ_i, X_t^i), (Θ_k, X_t^k) are i.i.d. for $k \neq i$.

3.1. Notation and Assumptions V

- For each portfolio, due to the assumption (H2), $\mu(\Theta_i)$ depends only on the observations and the linear credibility estimator is of the form

$$\mu(\Theta_i) = a_0^i + \sum_{t=1}^{T_i} a_t^i X_t^i,$$

where the coefficients a_t^i are estimated by minimizing the mean squared errors criterion.

- The estimation of the these parameters :

$$\widehat{a}_0^i = 1 - \frac{\tau^2 \omega_\bullet^i}{\sigma^2 + \tau^2 \omega_\bullet^i} \quad \text{and} \quad \widehat{a}_t^i = \frac{\tau^2 \omega_t^i}{\sigma^2 + \tau^2 \omega_\bullet^i}, \quad \text{with} \quad \omega_\bullet^i = \sum_{t=1}^{T_i} \omega_t^i \ .$$

3.1. Notation and Assumptions VI

- This leads to the following the Bühlmann-Straub credibility estimator of $X^i_{T_i+1}$

$$\widehat{X}^i_{T_i+1}(\Theta_i) = \alpha^i X^i_{\bullet} + (1-\alpha^i), \text{ with } \alpha^i = \omega^i_{\bullet}\tau^2/(\omega^i_{\bullet}\tau^2+\sigma^2),$$

where

- $X^i_{\bullet} = (\sum_{t=1}^{T_i} \omega^i_t X^i_t)/\omega^i_{\bullet}$
- σ^2/τ^2 represents the credibility coefficient.

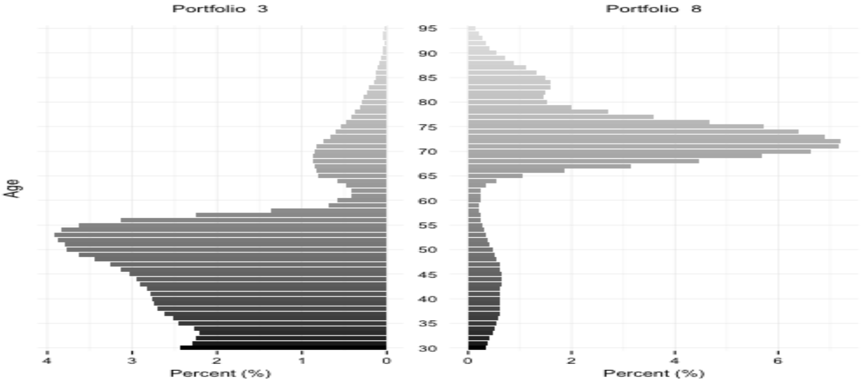
3.2. Application I

- The data come from studies conducted by *Institut des Actuaires*. These studies include in total **14 portfolios** covering the period **2007-2011** with each companies contributing data for at least 4 of a possible 5 years.
- The age band for all companies ranges from 30 to 95 years old.

Table 1: *Observed characteristics of portfolios population.*

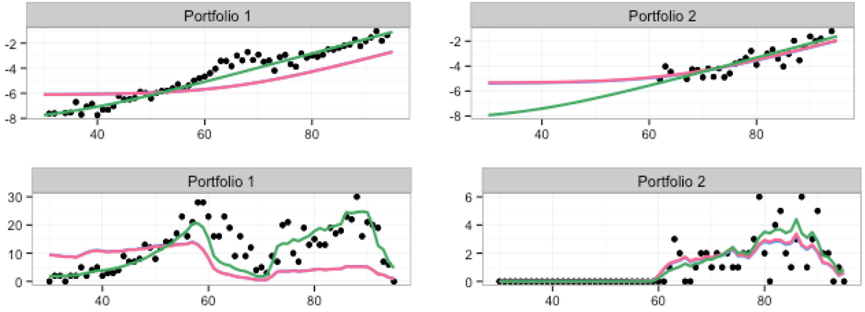
	Period of observation		Mean age		Average exposure	Mean age at death	Size
	Beginning	End	In	Out			
P1	01/01/07	12/31/11	36.96	39.74	2.77	68.78	616390
P2	01/01/07	12/31/11	69.3	73.35	4.05	80.34	7589
P3	01/01/07	12/31/10	40.16	43.1	2.94	71.77	80086
P4	01/01/07	12/31/11	37.5	41.13	3.63	54.08	93165
P5	01/01/07	12/31/11	36.9	39.1	2.2	59.31	21540
P6	01/01/07	12/31/10	48.5	52.11	3.62	82.34	847469
P7	01/01/07	12/31/11	66.65	71.29	4.64	73.68	89507
P8	01/01/07	04/13/11	67.51	71.38	3.86	80.72	78650
P9	01/01/07	06/30/11	45.97	49.6	3.62	73.17	1556150
P10	01/01/07	12/31/11	62.97	67.64	4.67	79.77	132990
P11	01/01/07	12/31/11	38.89	42	3.11	56.44	420405
P12	01/01/07	12/31/11	37.05	39.2	2.15	57.41	904020
P13	01/01/07	12/31/11	43.01	46.89	3.88	71.03	848757
P14	01/01/07	12/31/11	50.12	54.16	4.04	72.37	233488

3.2. Application II



3.2. Application III

- We fit the Makeham model for the baselines of mortality considered so as to estimate B_t^b for each calendar year while the parameters $A_t^b = A_T^b$ and $C_T^b = C_t^b$ remain fixed (for each t).
- We apply the credibility "updating" mechanism to predict the 2011 mortality based on the observations up to 2010



Semi-Parametric Approach : Local Likelihood Smoothing

4.1. Notation and Assumptions I

Assume that the intensity $\phi^{i}_{x,t}$ of the portfolio i is related to a reference $\phi^{\text{ref}}_{x,t}$ as follows :

$$\phi^{i}_{x,t} = \Theta^{x}_{i} \exp[f(x)]\phi^{\text{ref}}_{x,t},$$

where

- f is an unspecified, smooth and deterministic function of the age x.
- $\phi^{\text{ref}}_{x,t}$ is a reference

Goal

The approach is to first define an estimator of the quantity $\phi^{i}_{x,t}/\Theta^{x}_{i}$ then predict the unconditional force of mortality for each company i at the age level while imposing some assumptions on the risk behavior Θ^{x}_{i}.

4.1. Notation and Assumptions II

A1 The random vectors $\Theta^i = (\Theta^i_{x_1}, \cdots, \Theta^i_{x_n})$ are independent across companies and ages. Moreover, for $i = 1, \cdots, K$, Θ^i_x's are identically distributed with $\mathbb{E}[\Theta^i] = \mathbf{I}_n$ and $\mathbb{V}\text{ar}(\Theta^i) = \boldsymbol{\sigma}$, where $\boldsymbol{\sigma}$ is a diagonal matrix with elements σ_x and $\mathbf{I}_n$ is the identity matrix.

A2 The random vectors (φ^i, Θ^i), $i = 1, \cdots, K$, are independent across companies

A3 $\phi^i_{x_1}, \cdots, \phi^i_{x_n}$ are conditionally independent given Θ^i.

4.2. Procedure and Credibility Formula I

1. Estimation of $\exp[f(x)]\phi_x^{\text{ref}}$ using a local log-likelihood procedure given the information up to time T_i, where f is a polynomial function and denoted $\widehat{\phi}_x^i$ [1]

2. Then, the next period prediction of mortality curve ϕ^i (as a vector) is given by (Salhi and Thérond (2017))

$$\phi_{x,T_i+1}^i = \widetilde{\widetilde{\Theta}}_x^i \phi_x^{\text{ref}}$$

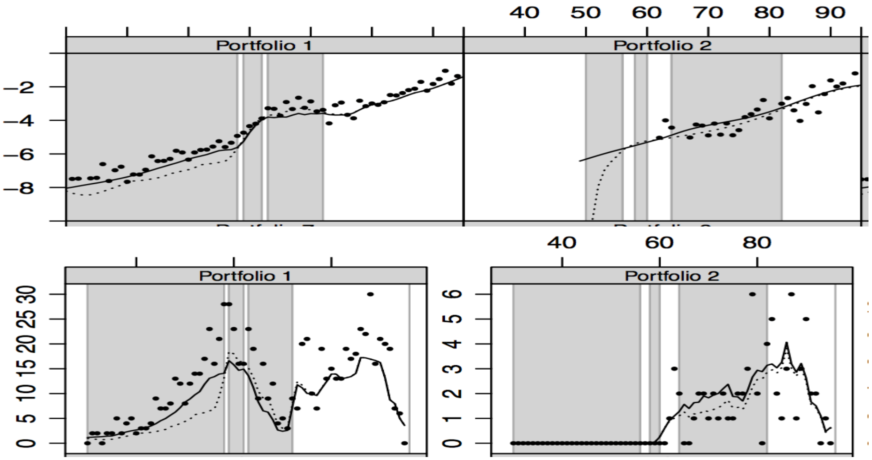
where

$$\widetilde{\widetilde{\Theta}^i} = \left(\mathbf{1}_n - \left(\phi^{\text{ref}}(\Sigma^i)^{-1}\sigma\phi^{\text{ref}}\right)^{\top}\right)\mathbf{1}_n + \left(\phi^{\text{ref}}(\Sigma^i)^{-1}\sigma\phi^{\text{ref}}\right)^{\top}\widehat{\Theta}^i,$$

such that $\Sigma^i = \mathbb{V}\text{ar}(\widehat{\phi}^i)$ and $\widehat{\Theta}_x^i = \widehat{\phi}_x^i/\phi_x^{\text{ref}}$.

1. i.e. local polynomial mortality estimation of Nielsen and Tanggaard (2001), Fan and Gijbels (1996) used also in Tomas et al. (2014)

4.3. Application I



4.3. Application II

Table 3: *Tests and quantities summarizing the deviation between the observations and the model*

		Hardy-Panjer	Poisson-Gamma	Makeham-Credibility	Smoothed	Smoothed+Adj.
χ^2	Portfolio 1	1901.240	1928.680	259.400	357.870	193.967
MAPE (%)	Portfolio 1	102.660	102.000	32.870	3.018	2.349
SMR	Portfolio 1	1.737	1.756	1.126	1.487	1.385
χ^2	Portfolio 2	34.890	33.640	30.940	37.612	31.166
MAPE (%)	Portfolio 2	48.030	49.120	53.990	20.119	20.842
SMR	Portfolio 2	1.037	1.002	0.905	1.102	0.948

Main References

Bühlmann, H. and Gisler, A. (2005). *A course in credibility theory and its applications*. Springer.

Fan, J. and Gijbels, I. (1996). *Local Polynomial Modelling and Its Applications*. Monographs on Statistics and Applied Probability 66. Chapman and Hall.

Hardy, M. R. and Panjer, H. H. (1998). A credibility approach to mortality risk. *ASTIN Bulletin*, 28(2) :269–283.

Nielsen, J. and Tanggaard, C. (2001). Boundary and bias correction in kernel hazard estimation. *Scandinavian Journal of Statistics*, vol. 28(4) :675–698.

Planchet, F. and Thérond, P. (2011). *Modélisation statistique des phénomènes de durée - Applications actuarielles*. Assurance Audit Actuariat. Economica Paris.

Salhi, Y., Thérond, P.-E., and Tomas, J. (2016). A credibility approach of the makeham mortality law. *European Actuarial Journal*, 6(1) :61–96.

Tomas, J., Planchet, F., et al. (2014). Constructing entity specific projected mortality table : adjustment to a reference. *European Actuarial Journal*, 4(2) :247–279.