Some characteristics of an equity security next-year impairment

BNP Paribas Cardif. Séminaire technique.

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joint work with P. Therond et S. Loisel

Plan

Plan

Some figures

(Mds €)	Allianz	Axa	CNP Assurances	Generali
Balance Sheet Size	641.472	730.085	321.011	423.057
Total equity	47.253	50.932	13.217	18.120
AFS Assets	333.880	355.126	231.709	175.649
AFS (Funds and equity securities)	26.188	20.636	27.618	20.53

(Mds €)	Allianz	Axa	CNP Assurances	Generali
Result	2804	4516	1141	1153
Impairment losses on AFS funds and equity securities	-2487	-860	-1600	-781

- Non negligible element. How to control them?

IAS 39; §55

a gain or loss on an AFS financial asset shall be recognized in OCI, except for impairment losses and foreign exchange gains and losses, until the financial asset is de-recognized

IAS 39; §59

A financial asset (...) is impaired and impairment losses are incurred if, and only if, there is objective evidence of impairment as a result of one or more events that occurred after the initial recognition of the asset (a "loss event") and that loss event (or events) has an impact on the estimated future cash flows of the financial asset (...) that can be reliably estimated.

IAS 39; §61

A **significant or prolonged decline** *in the fair value of an investment in an equity instrument below its cost is also objective evidence of impairment.*

IAS 39 impairment disposals

Category	HTM	AFS		HFT
Eligible securities	Bonds	Bonds	Others (stock, funds, etc.)	Everything
Valuation	Amortized cost	Fair Value (through OCI)		Fair Value through P&L
Impairment principle	Event of proven loss	Event of proven loss	Significant or prolonged fall in the fair value	NA
Impairment trigger	Objective evidence resulting from an incurred event (cf. IAS 39 §59)		**Two critera (non-cumulative; Cf. IFRIC Update of July 2009) : significant or prolonged loss in the FV**	NA
Impairment Value	Difference between the amortized cost and the revised value of future flows discounted at the original interest rate	In **result** : difference between reported value (before impairment) and the FV		NA
Reversal of the impairment	Possible in specific cases	Possible in specific cases	**Impossible**	NA

Figures : BNP Paribas from 2010 until 2012

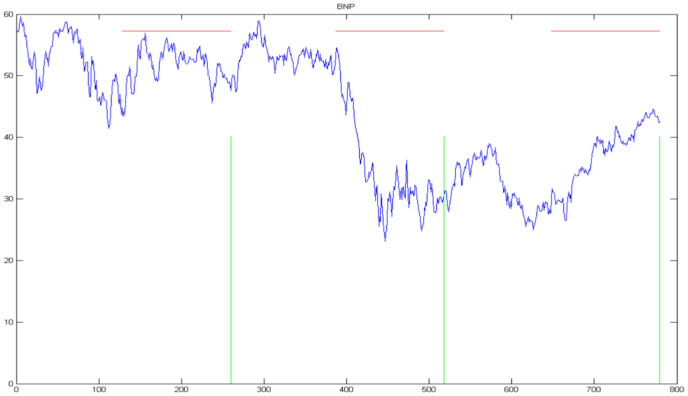


Figure: $\alpha = 0.3$, $s = 0.5y$

Figures : Total from 2010 until 2012 ...

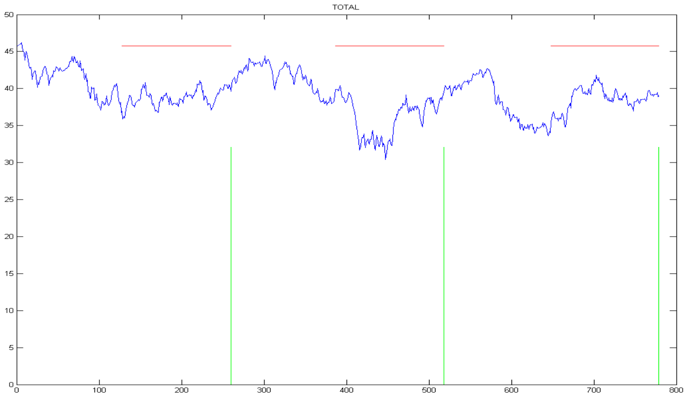


Figure: $\alpha = 0.3$, $s = 0.5y$

... with impairments

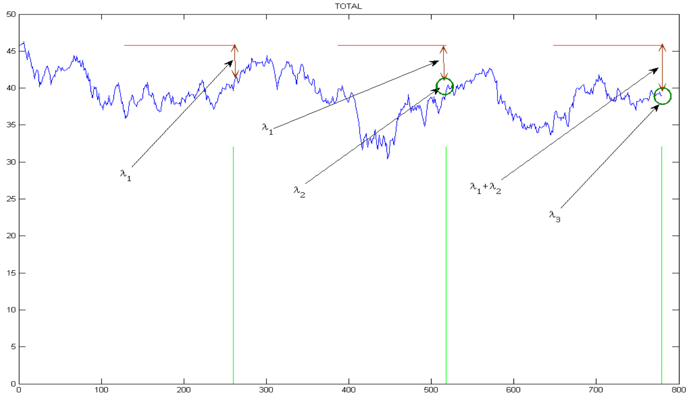


Figure: $\alpha = 0.3$, $s = 0.5y$

Model : $\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$

Impairment criterion

$$\begin{cases} S_{t+1} \leq (1-\alpha) S_{t_a}, \text{or;} \\ \forall u \in \left]t+1-s, t+1\right], S_u \leq S_{t_a}, \end{cases}$$

And

$$S_{t+1} \leq S_{t_a} - \Lambda_t = K_t.$$

Impairment value

$$\lambda_t = (K_t - S_{t+1})^+ \mathbf{1}\left\{ \max_{t+1-s \leq u \leq t+1} S_u \leq S_{t_a} \cup S_{t+1} \leq (1-\alpha) S_{t_a} \right\}.$$

- Use financial tools.

Plan

Theorem (Impairment probability)

$$\mathbb{P}_t\left[J_{t+1}\right] = \left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\left[\Psi_\rho\left(C, D(K_t)\right) - \Psi_\rho\left(C, D(m_t)\right)\right] + \Phi\left(-A(K_t)\right) + \Psi_\rho\left(B, A(K_t)\right) - \Psi_\rho\left(B, A(m_t)\right), \quad (1)$$

where, for $x \in \{m_t, K_t\}$,

- $A(x) = \frac{\ln(S_t/x)+\mu}{\sigma} - \frac{\sigma}{2}$, $A'(x) = A(x) + \sigma$,
- $B = \frac{\ln(S_t/S_{t_a})+\mu(1-s)}{\sigma\sqrt{(1-s)}} - \frac{\sigma\sqrt{(1-s)}}{2}$, $B' = B + \sigma\sqrt{(1-s)}$,
- $C = \frac{\ln(S_{t_a}/S_t)+\mu(1-s)}{\sigma\sqrt{(1-s)}} - \frac{\sigma\sqrt{(1-s)}}{2}$, $C' = C + \sigma\sqrt{(1-s)}$,
- $D(x) = \frac{\ln(S_{t_a}^2/S_t x)+\mu}{\sigma} - \frac{\sigma}{2}$, $D'(x) = D(x) + \sigma$,
- $k_1 = \frac{2\mu}{\sigma^2}$,

Φ denotes the c.d.f. of a standard normal distribution, and Ψ_ρ is the bivariate normal distribution function: for all x, y, $\Psi_\rho(x, y) = \mathbb{P}_t\left[X \le x, Y \le y\right]$ where (X, Y) is a Gaussian vector with standard marginals and correlation ρ, $\rho = \sqrt{1-s}$.

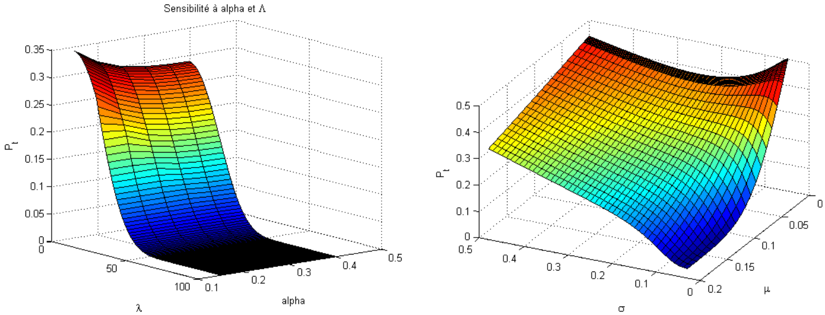
Sketch of proof

Joint law of a drifted BM and its maximum

Proposition

$$\mathbb{P}_t\left[\max_{t+1-s\leq u\leq t+1} B_u \geq a,\ B_{t+1}\leq z\right] =$$
$$\exp\left(\frac{2(\mu-\frac{\sigma^2}{2})a}{\sigma^2}\right)\left[\Phi\left(\frac{-2a+z-\mu}{\sigma}+\frac{\sigma}{2}\right)\right.$$
$$\left.-\Psi_\rho\left(\frac{-a-(1-s)\mu}{\sigma\sqrt{1-s}}-\frac{\sigma\sqrt{1-s}}{2},\frac{-2a+z-\mu}{\sigma}+\frac{\sigma}{2})\right)\right] \tag{2}$$
$$+\left[1-\Phi\left(\frac{a-(1-s)\mu}{\sigma\sqrt{1-s}}+\frac{\sigma\sqrt{1-s}}{2}\right)\right]$$
$$-\Psi_\rho\left(\frac{-a+(1-s)\mu}{\sigma\sqrt{1-s}}-\frac{\sigma\sqrt{1-s}}{2},\frac{-z+\mu}{\sigma}-\frac{\sigma}{2}\right).$$

Figures : some sensitivities



Sensitivities w.r.t. Λ_t and α (left), to σ and μ (right).
Central scenario : $S_{t_a} = 100$, $S_t = 95$, $\mu = \ln(1.08)$, $\sigma = 0.25$, $\Lambda_t = 5$, $\alpha = 0.3$, $s = 0.5$.

Theorem (Impairments expectation)

$$
\begin{aligned}
\mathbb{E}_t\left[\lambda_{t+1}\right] = {} & S_t e^{\mu}\left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\left[\Psi_{-\rho}\left(C', -D'(K_t)\right) - \Psi_{-\rho}\left(C', -D'(m_t)\right)\right] \\
& + S_t e^{\mu}\left[\Psi_{\rho}\left(-B', -A'(m_t)\right) - \Phi\left(-A'(m_t)\right) - \Psi_{\rho}\left(-B', -A'(K_t)\right)\right] \\
& + K_t\left[\Psi_{\rho}\left(-B, -A(K_t)\right) + \Phi\left(-A(m_t)\right) - \Psi_{\rho}\left(-B, -A(m_t)\right)\right] \\
& + (K_t - m_t)\left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\left[\Phi\left(-D(m_t)\right) - \Psi_{\rho}\left(-C, -D(m_t)\right)\right] \\
& - K_t\left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\Psi_{-\rho}\left(C, -D(K_t)\right) \\
& + m_t\left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\Psi_{-\rho}\left(C, -D(m_t)\right).
\end{aligned} \tag{3}
$$

Sketch of proof

Decomposition

The payoff we are interested in is :
$\lambda_{t+1} = (K_t - S_{t+1})^+ \mathbf{1}\{\max_{t+1-s \le u \le t+1} S_u \le S_{t_a} \cup S_{t+1} \le (1-\alpha) S_{t_a}\}$,
with $K_t = S_{t_a} - \Lambda_t$. On a :

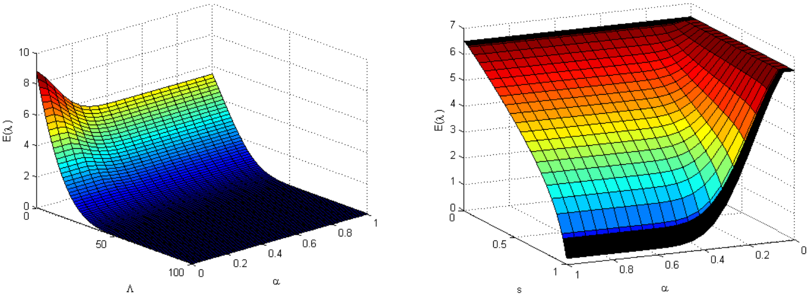
$$\lambda_{t+1} = X_{t+1} + Y_{t+1} - Z_{t+1},$$

with

- $X_{t+1} = (K_t - S_{t+1})^+ \mathbf{1}\{\max_{t+1-s \le u \le t+1} S_u \le S_{t_a}\}$;
- $Y_{t+1} = (K_t - S_{t+1})^+ \mathbf{1}\{S_{t+1} \le (1-\alpha) S_{t_a}\}$;
- $Z_{t+1} =$ $(K_t - S_{t+1})^+ \mathbf{1}\{\max_{t+1-s \le u \le t+1} S_u \le S_{t_a}\} . \mathbf{1}\{S_{t+1} \le (1-\alpha) S_{t_a}\}$.

▶ Barrier options : *rear-end up-and-out put*

Figures : some sensitivities



Sensitivities w.r.t. Λ_t and α (left), to s and α (right).
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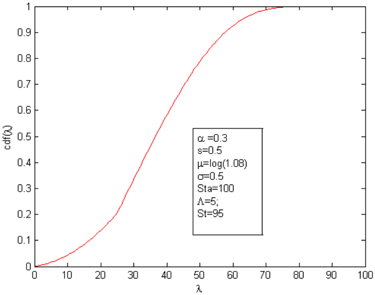
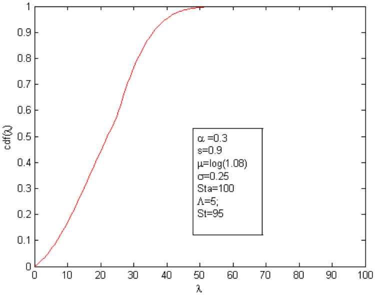
Theorem (C.D.F. of impairments)

The cumulative distribution function of the next-year impairments, given the information $\mathcal{F}_t$ at time t, is given by

$$\mathbb{P}_t\left[\lambda_{t+1} \leq l\right] = \begin{cases} \Phi\left(A(K_t - l)\right) + \Psi_\rho\left(B, A(m_t)\right) - \Psi_\rho\left(B, A(K_t - l)\right) \\ + \left(\frac{S_{t_a}}{S_t}\right)^{k_1 - 1}\left[\Psi_\rho\left(C, D(m_t)\right) - \Psi_\rho\left(C, D(K_t - l)\right)\right] & , 0 \leq l \leq K_t - m_t, \\ \Phi\left(A(K_t - l)\right) & , K_t - m_t < l \leq K_t. \end{cases} \tag{4}$$

Two examples





Plan

Datas

	S_{t_a} (€)	Annual volatility	Arbitrary μ
BNP Paribas	57.24	48.92%	14.92%
Pernod Ricard NV	60.83	22.80%	5.56%
Bouyges	37.02	31.56%	7.94%
Carrefour	29.7016	33.80%	8.67%
Total	45.795	22.61%	5.51%

Impairments for the year :	2010	2011	2012	Total
BNP Paribas	9.63	17.26	0	26.89
Pernod Ricard NV	0	0	0	0
Bouyges	4.76	7.92	1.94	14.62
Carrefour	0	12.0916	0	12.0916
Total	6.145	0.15	0.49	6.785

BNP

BNP Paribas. $S_{t_a} = 57.24$

S_t	Λ_t	$\mathbb{P}_t\left[J_{t+1}\right]$	$\mathbb{E}_t\left[\lambda_{t+1}\right]$	$VaR(80\%)$	$VaR(95\%)$	$VaR(99.5\%)$
42.58	28.62	0.1915	1.1968	0	9.0056	16.1811
	42.93	0.011	0.0229	0	0	1.8711

Bouyges and Carrefour

Bouyges. $S_{t_a} = 37.02$

S_t	Λ_t	$\mathbb{P}_t[J_{t+1}]$	$\mathbb{E}_t[\lambda_{t+1}]$	$VaR(80\%)$	$VaR(95\%)$	$VaR(99.5\%)$
22.4	18.51	0.2425	0.722	0.8192	4.7805	8.2759
	27.765	0.0019	0.0015	0	0	0

Carrefour. $S_{t_a} = 29.7016$

S_t	Λ	$\mathbb{P}_t[J_{t+1}]$	$\mathbb{E}_t[\lambda_{t+1}]$	$VaR(80\%)$	$VaR(95\%)$	$VaR(99.5\%)$
19.34	14.8508	0.1924	0.4592	0	3.4259	6.5103
	22.2762	0.0018	0.0012	0	0	0

Total and Pernod Ricard

Total. $S_{t_a} = 45.795$

S_t	Λ_t	$\mathbb{P}_t[J_{t+1}]$	$\mathbb{E}_t[\lambda_{t+1}]$	$VaR(80\%)$	$VaR(95\%)$	$VaR(99.5\%)$
38.42	2.28975	0.5509	4.0699	10.7697	16.2236	21.402
	4.5795	0.5075	3.3007	8.4799	13.9338	19.1123
	22.8975	0.0078	0.0124	0	0	0.7943
	34.34625	0	0	0	0	0

Pernod Ricard. $S_t = 49.26$ at $t = 12/31/2008$

S_{t_a}	Λ_t	$\mathbb{P}_t[J_{t+1}]$	$\mathbb{E}_t[\lambda_{t+1}]$	$VaR(80\%)$	$VaR(95\%)$	$VaR(99.5\%)$
41.98	0	0.0912	1.8944	0	10.9932	19.3704
71.60	22.34	0.4625	4.5728	10.2986	18.9791	26.6504
44.53	0	0.1349	2.3653	0	14.2491	21.9204

Table: Results for Pernod Ricard, with $\sigma = 31.38\%$ and $\mu = 7.88\%$, in different scenarios.

Plan

What to solve?

New expectation

$$\begin{aligned}\mathbb{E}_t\left[\widetilde{\lambda}_{t+1}^{\alpha_1}\right] &= \mathbb{E}_t\left[(K_t - S_{t+1})^+ \mathbf{1}\left\{S_{t+1} \le (1-\alpha_1)S_{t_a}\right\}\right] \\ &= -S_t e^{\mu}\Phi\left(-A'(m_t^{\alpha_1})\right) + K_t\Phi\left(-A(m_t^{\alpha_1})\right),\end{aligned}$$

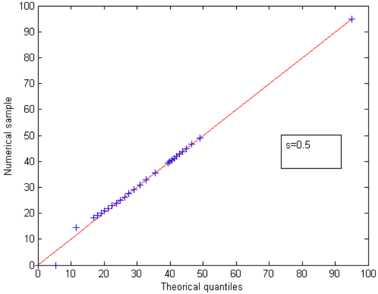
$m_t^{\alpha_1} = \min\left(K_t, (1-\alpha_1)S_{t_a}\right)$.

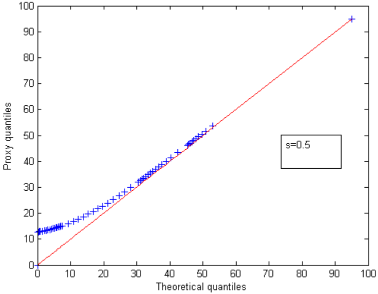
Or :

$$\mathbb{E}_t\left[\widetilde{\lambda}_{t+1}^{\alpha_1}\right] = \mathbb{E}_t\left[\lambda_{t+1}\right] \tag{5}$$

- As for the implied volatility.

QQ-plots





Left : QQ-plot for proxy distribution (Y-axis) and "classical" distribution (X-axis).
Right : QQ-plot for r.v.'s conditioned to be positive.

Quantiles

$\alpha_0 = 0.3$	$s = 1/3$	$s = 0.5$	$s = 0.75$
$q_{\alpha_0,s}(0.05)$	2.88	3.20	3.65
$q_{\alpha_1}(0.05)$	16.82	16.82	16.82
$q_{\alpha_0,s}(0.5)$	17.09	18.25	20.15
$q_{\alpha_1}(0.5)$	24.05	24.05	24.05
$q_{\alpha_0,s}(0.95)$	36.89	37.66	38.84
$q_{\alpha_1}(0.95)$	38.83	38.83	38.83
$q_{\alpha_1}(p) - q_{\alpha_0,s}(p) < 0.01$	$p \geq 0.85$	$p \geq 0.75$	$p \geq 0.5$

Quantiles of classical distribution $(q_{\alpha_0,s}(.))$ and proxy distribution $(q_{\alpha_1}(.))$

- Very easy to use
- Relevant for big Λ_t
- Applicable in a few cases

Plan

Brownian bridges

$$X_t = x_0 \exp\left((\mu - \frac{\sigma^2}{2}) T + \sigma \sqrt{T} Z \right), \; Z \sim \mathcal{N}(0,1).$$

Conditionally to x_0 et x_T, one has :

$$\mathbb{P}\left[\sup_{[0,T]} X > H \right] = \begin{cases} 1 & \text{si } x_0 \text{ ou } X_T > H \\ \exp\left(-2 \frac{(X_T - U)(x_0 - U)}{x_0^2 \sigma^2} \right) & \text{sinon.} \end{cases} \quad (6)$$

- Monte-Carlo based algorithm
- Better results than a "naive" approach

Issues

- **Slow** convergence
- "Edges" issues
- Over estimation

THANK YOU

Some references

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